Becoming a Killer A model of prey localisation and recognition by means of the lateral line

Martin Lingenheil

Physik Department TU München

July 8, 2004



Martin Lingenheil

Becoming a Killer

・ 同 ト・ ・ ヨート・ ・ ヨート

Agenda

Introduction Minimal Model Field Reconstruction Neuronal Implementation of the Model

Introduction

- Xenopus and its Lateral Line
- Water Waves

2 Minimal Model

- Reconstruction of the Wave Form
- Performance of the Minimal Model
- Learning

3 Field Reconstruction

- Full-Field Reconstruction
- Window of Reference

4 Neuronal Implementation of the Model

- Implementing a Convolution
- Learning a Neuronal Model
- Performance of a Neuronal Model

Xenopus and its Lateral Line Water Waves

Xenopus laevis laevis

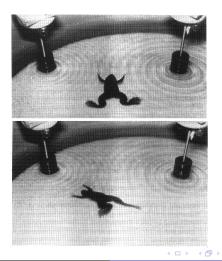


- completely aquatic
- carnivorous
- nocturnal
- lives in turbid waters
- prey localisation by means of the lateral line

・ロト ・ 一下・ ・ 三ト・・

Xenopus and its Lateral Line Water Waves

Behavioural Experiments



Martin Lingenheil

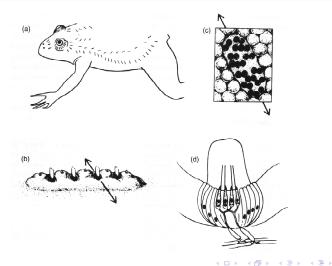
Becoming a Killer

→ < Ξ >

Xenopus and its Lateral Line Water Waves

4

Lateral Line System



Performance

Xenopus and its Lateral Line Water Waves

3

• robust with respect to most lesions

Xenopus and its Lateral Line Water Waves

イロト イヨト イヨト イヨト

4

Performance

- robust with respect to most lesions
- robust with respect to body position

Xenopus and its Lateral Line Water Waves



- robust with respect to most lesions
- robust with respect to body position
- wave source localisation ($\geq 5^{\circ}$ accuracy)

(日) (日) (日) (日) (日)

æ

Xenopus and its Lateral Line Water Waves

Performance

- robust with respect to most lesions
- robust with respect to body position
- wave source localisation ($\geq 5^{\circ}$ accuracy)
- frequency discrimination and recognition (approx. 4% discrimination threshold)

Xenopus and its Lateral Line Water Waves

▲□► ▲ □► ▲

Performance

- robust with respect to most lesions
- robust with respect to body position
- wave source localisation ($\geq 5^{\circ}$ accuracy)
- frequency discrimination and recognition (approx. 4% discrimination threshold)
- discrimination of the superposition of both waves

Xenopus and its Lateral Line Water Waves

Performance

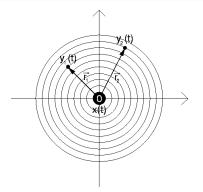
- robust with respect to most lesions
- robust with respect to body position
- wave source localisation ($\geq 5^{\circ}$ accuracy)
- frequency discrimination and recognition (approx. 4% discrimination threshold)
- discrimination of the superposition of both waves
- normal operating range: \leq 30 Hz

Xenopus and its Lateral Line Water Waves

・ロト ・御 ト ・ モト ・ モト

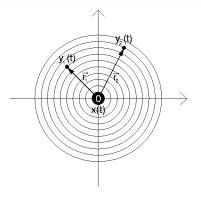
æ

Water Waves I



Xenopus and its Lateral Line Water Waves

Water Waves I



$$y_{1,2}(t) = \int_{-\infty}^{+\infty} h(t - \tau, \mathbf{r}_{1,2}) x(\tau) d\tau$$
 (1)

・ロト ・御 ト ・ モト ・ モト

æ

Xenopus and its Lateral Line Water Waves

Water Waves II

Fourier transformed with respect to time the equation looks like this

$$Y_{1,2}(\omega) = H(\omega, \mathbf{r}_{1,2})X(\omega) , \qquad (2)$$

イロト イヨト イヨト イヨト

-2

with the transfer function

$$H(\omega, \mathbf{r}) = \sqrt{\frac{r_0}{r}} 10^{-2|\Delta\varphi|/\pi} \times \exp\left[-\frac{4\nu k^3}{3\omega} (r - r_0) - kd - ik (r - r_0)\right] \quad (3)$$
$$\omega^2 = \left(gk + \frac{Tk^3}{\rho}\right) \tanh(Dk) \quad (4)$$

Xenopus and its Lateral Line Water Waves

A (10) < 10</p>

Impulse Response

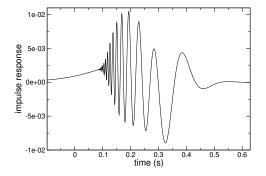


Figure: Impulse response at the position of an organ at 8 cm distance from the source.

Xenopus and its Lateral Line Water Waves

・ロト ・ 日ト ・ モト

- ∢ ≣ →

4

Transfer Function

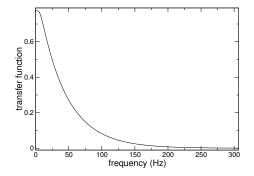


Figure: Same situation as on the last slide: Transfer function.

Reconstruction of the Wave Form Performance of the Minimal Model Learning

▲□ ▶ - ▲ 三 ▶ -

Introduction

- Xenopus and its Lateral Line
- Water Waves

2 Minimal Model

- Reconstruction of the Wave Form
- Performance of the Minimal Model
- Learning

3 Field Reconstruction

- Full-Field Reconstruction
- Window of Reference
- 4 Neuronal Implementation of the Model
 - Implementing a Convolution
 - Learning a Neuronal Model
 - Performance of a Neuronal Model

Reconstruction of the Wave Form Performance of the Minimal Model Learning

イロト イヨト イヨト イヨト

3

Outline of the Model

stimulus localisation

Reconstruction of the Wave Form Performance of the Minimal Model Learning

(ロ) (同) (注) (注)

-2

Outline of the Model

- stimulus localisation
- hypothesis: reconstruction of the wave form

Reconstruction of the Wave Form Performance of the Minimal Model Learning

イロト イポト イヨト イヨト

-2

Outline of the Model

- stimulus localisation
- hypothesis: reconstruction of the wave form
- robust with respect to noise

Reconstruction of the Wave Form Performance of the Minimal Model Learning

<ロ> (日) (日) (日) (日) (日)

-1

Outline of the Model

- stimulus localisation
- hypothesis: reconstruction of the wave form
- robust with respect to noise

 \Rightarrow Construct a maximum-likelihood estimator for the original wave form!

Reconstruction of the Wave Form Performance of the Minimal Model Learning

Wave Form Reconstruction I

Create an estimator $\hat{x}(t)$ for the original wave form x(t) by minimizing expectation of the error

$$\bar{E} = \left\langle \int \left[\hat{x}(t) - x(t) \right]^2 dt \right\rangle , \qquad (5)$$

where \hat{x} can be written as

$$\hat{x} = \sum_{j} s_{j} \star y_{j} , \qquad (6)$$
$$y_{j} = h_{j} \star x . \qquad (7)$$

(日) (日) (日) (日) (日)

-1

Reconstruction of the Wave Form Performance of the Minimal Model Learning

<ロ> (四) (四) (三) (三) (三)

-2

Wave Form Reconstruction II

Find the minimum of \bar{E} with respect to the model parameters

$$\frac{\delta \bar{E}}{\delta s_j(\tau)} = 2 \left\langle \int \left\{ \sum_i \left[\mathbf{s}_i \star y_i \right](t) - x(t) \right\} y_j(t-\tau) dt \right\rangle \stackrel{!}{=} 0 , \quad (8)$$

Reconstruction of the Wave Form Performance of the Minimal Model Learning

Wave Form Reconstruction II

Find the minimum of \bar{E} with respect to the model parameters

$$\frac{\delta \bar{E}}{\delta s_j(\tau)} = 2 \left\langle \int \left\{ \sum_i \left[s_i \star y_i \right](t) - x(t) \right\} y_j(t-\tau) dt \right\rangle \stackrel{!}{=} 0 , \quad (8)$$

which looks quite plain if Fourier transformed

$$0 = \left\langle \left(\sum_{i} S_{i} Y_{i} - X \right) Y_{j}^{*} \right\rangle .$$
(9)

イロト イポト イヨト イヨト

-2

Reconstruction of the Wave Form Performance of the Minimal Model Learning

Wave Form Reconstruction II

Find the minimum of \bar{E} with respect to the model parameters

$$\frac{\delta \bar{E}}{\delta s_j(\tau)} = 2 \left\langle \int \left\{ \sum_i \left[s_i \star y_i \right](t) - x(t) \right\} y_j(t-\tau) dt \right\rangle \stackrel{!}{=} 0 , \quad (8)$$

which looks quite plain if Fourier transformed

$$0 = \left\langle \left(\sum_{i} S_{i} Y_{i} - X \right) Y_{j}^{*} \right\rangle .$$
(9)

Assuming the noise to be Gaussian and with $\sigma:=\frac{\sigma_n}{\sigma_x}$ this Eq. is solved by

$$S_{j} = \frac{H_{j}^{*}}{\sum_{i} |H_{i}|^{2} + \sigma^{2}} .$$
 (10)

Reconstruction of the Wave Form Performance of the Minimal Model Learning

A (1) < (1) < (1) < (1) </p>

Reverse Transfer Function I

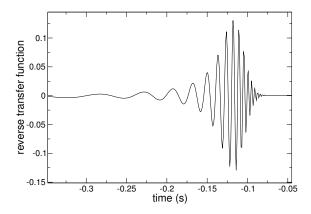


Figure: Reverse transfer function for an organ at 8 cm from the source (time domain).

Reconstruction of the Wave Form Performance of the Minimal Model Learning

▲御▶ ▲ 副▶

≣⇒

4

Reverse Transfer Function II

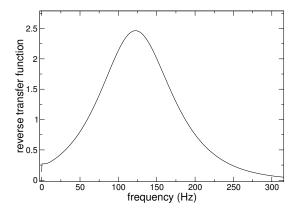


Figure: Same as the last slide: frequency domain.

Reconstruction of the Wave Form Performance of the Minimal Model

æ

Reconstruction of a Monofrequent Stimulus

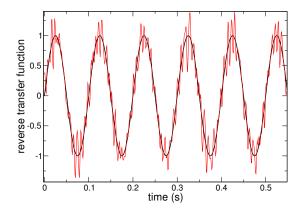


Figure: Reconstruction of the wave form using equation (7) (f = 10 Hz, $\sigma_x = 1, \ \sigma_n = 0.1$). (日) (部) (注) (注)

Reconstruction of the Wave Form Performance of the Minimal Model Learning

Reconstruction of a Complex Stimulus

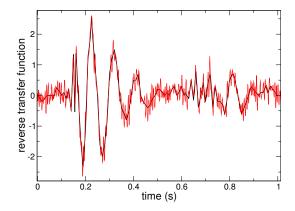


Figure: Same as on the last slide, but the stimulus is more realistic.

Reconstruction of the Wave Form Performance of the Minimal Model Learning

Localisation I

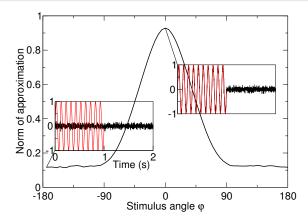


Figure: Map of reconstruction norms for various assumed source angles $(f = 10 \text{ Hz}, \sigma_x = 0.5, \sigma_n = 0.1)$

Reconstruction of the Wave Form Performance of the Minimal Model Learning

イロト イヨト イヨト イヨト

4

Localisation II

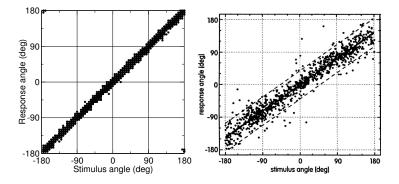


Figure: Localisation in model and experiment.

Reconstruction of the Wave Form Performance of the Minimal Model Learning

Wave Source Discrimination

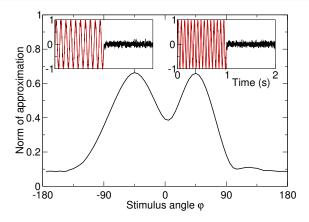


Figure: Two sources of 10 and 15 Hz, respectively, can be clearly discriminated.

Reconstruction of the Wave Form Performance of the Minimal Model Learning

Learning Algorithm

- return to the gerneralised gradient of the expected error (equ. (8))
- online learning rule to find the minimum: descent along the gradient

$$\Delta s_j(\tau) = -\eta \frac{\delta E}{\delta s_j(\tau)}$$
(11)
= $\mathcal{F}^{-1} \left[-2\eta \left(\sum_i S_i Y_i - X \right) Y_j^* \right]$ (12)

(日) (部) (注) (注)

Reconstruction of the Wave Form Performance of the Minimal Model Learning

・ロト ・ 一下・ ・ 三ト・・

3

Results I

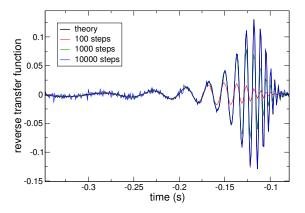


Figure: Linear learning: convergence ($\sigma = 0.1$, $\sigma_n = 0.1$, $\eta = 10^{-5}$).

Reconstruction of the Wave Form Performance of the Minimal Model Learning

・ロト ・ 日 ・ ・ 目 ト

Э

4

Results II

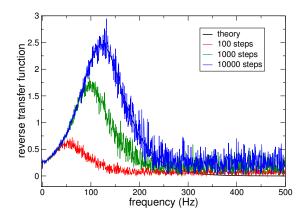


Figure: Convergence in frequency space.

Full-Field Reconstruction Window of Reference

Introduction

- Xenopus and its Lateral Line
- Water Waves

2 Minimal Model

- Reconstruction of the Wave Form
- Performance of the Minimal Model
- Learning

3 Field Reconstruction

- Full-Field Reconstruction
- Window of Reference
- 4 Neuronal Implementation of the Model
 - Implementing a Convolution
 - Learning a Neuronal Model
 - Performance of a Neuronal Model

・日・ ・ ヨ・ ・

Full-Field Reconstruction Window of Reference

Full-Field Theory I

We want to

• improve the localisation performance,

イロト イヨト イヨト イヨト

-2

Full-Field Reconstruction Window of Reference

Full-Field Theory I

We want to

- improve the localisation performance,
- simplify the implementation of the learning process.

(日) (日) (日) (日) (日)

æ

Full-Field Reconstruction Window of Reference

Full-Field Theory I

We want to

- improve the localisation performance,
- simplify the implementation of the learning process.

 \Rightarrow Find the maximum likelihood reconstruction with respect to the whole field!

$$0 \stackrel{!}{=} \frac{\delta \bar{E}_{\rm ff}}{\delta s_j^{\mathbf{p}}(\tau)}$$
(13)
$$\bar{E}_{\rm ff} = \left\langle \sum_{\mathbf{p}} \int_{-\infty}^{+\infty} \left[x^{\mathbf{p}}(t) - \hat{x}^{\mathbf{p}}(t) \right]^2 dt \right\rangle$$
(14)

< ロト (周) (日) (日)

Full-Field Reconstruction Window of Reference

Full-Field Theory II

It can be shown that $\hat{x}^{\mathbf{p}}$ is again found by convolution of the signal coming from the organs with reverse transfer fundtions

$$\hat{x}^{\mathbf{p}} = \sum_{i} s_{i}^{\mathbf{p}} \star y_{i} .$$
(15)

イロト イポト イヨト イヨト

 \bar{E}_{ff} is minimised if $S^{\mathbf{p}}_{j}(\omega)$ solves the following system of linear equations:

$$\sum_{\mathbf{p}} \left[\sum_{i} H_{i}^{\mathbf{q}*}(\omega) H_{i}^{\mathbf{p}}(\omega) + \sigma^{2} \delta_{\mathbf{q}\mathbf{p}} \right] S_{j}^{\mathbf{p}}(\omega) = H_{j}^{\mathbf{q}*}(\omega).$$
 (16)

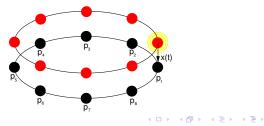
Full-Field Reconstruction Window of Reference

Full-Field Learning

To learn this, we can use something like equation (12) again

$$\Delta s_{j}^{\mathbf{p}'}(\tau) = -\eta \frac{\delta E_{\mathrm{ff}}}{\delta s_{j}^{\mathbf{p}'}(\tau)}$$

$$= \mathcal{F}^{-1} \left[-2\eta \left(\sum_{i} S_{i}^{\mathbf{p}'} Y_{i} - \delta_{\mathbf{p},\mathbf{p}'} X^{\mathbf{p}} \right) Y_{j}^{*} \right]$$
(18)



æ

Full-Field Reconstruction Window of Reference

Full-Field Calculation

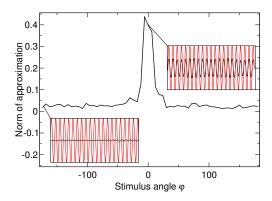


Figure: Map of reconstructions with a learned model (60 organs, 200000 steps).

・ロト ・日下 ・ヨト ・

э

Full-Field Reconstruction Window of Reference

Convergence

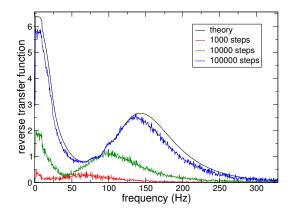


Figure: Full-field learning session (60 units).

▲ 御 ▶ → ● ●

Full-Field Reconstruction Window of Reference

Problems with Full-Field Learning

very slow convergence

イロト イヨト イヨト イヨト

-2

Full-Field Reconstruction Window of Reference

Problems with Full-Field Learning

- very slow convergence
- individual reconstruction norm strongly influcenced by stochastic inhomogeneities during learning

・ 同 ト・ ・ ヨート・ ・ ヨート

Full-Field Reconstruction Window of Reference

Problems with Full-Field Learning

- very slow convergence
- individual reconstruction norm strongly influcenced by stochastic inhomogeneities during learning
- improvement of localisation at the expense of reconstruction quality

▲□► ▲ □► ▲

Full-Field Reconstruction Window of Reference

Problems with Full-Field Learning

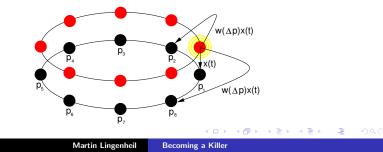
- very slow convergence
- individual reconstruction norm strongly influcenced by stochastic inhomogeneities during learning
- improvement of localisation at the expense of reconstruction quality
- \Rightarrow Find a compromise between single position and full-field reconstruction!

Full-Field Reconstruction Window of Reference

Window of Reference I

The idea is to give reference input $x^{\mathbf{p}}(t)$ to a unit assuming input at position \mathbf{p}' weighted by the window of reference:

$$\Delta s_j^{\mathbf{p}'}(\tau) = \mathcal{F}^{-1}\left[-2\eta \left(\sum_i S_i^{\mathbf{p}'} Y_i - W\left(\Delta \mathbf{p}\right) X^{\mathbf{p}}\right) Y_j^*\right]$$
(19)

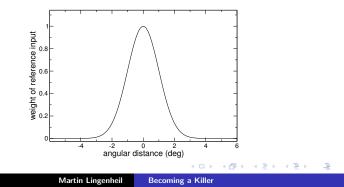


Full-Field Reconstruction Window of Reference

Window of Reference II

An example is be the gaussian function:

$$W\left(\mathbf{p}'-\mathbf{p}\right) = e^{-\frac{1}{2}\frac{\Delta\varphi^2}{\sigma_{\varphi}^2}}$$
(20)



Full-Field Reconstruction Window of Reference

Learning a WOR model

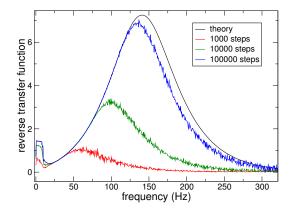


Figure: Convergence of a WOR model ($\sigma_{\varphi} = 12^{\circ}$).

(日) (部) (注) (注)

Full-Field Reconstruction Window of Reference

Performance of a WOR Model

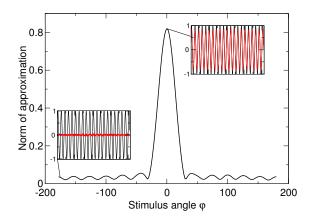


Figure: Localisation performance of a WOR model ($\sigma_{\varphi} = 10^{\circ}$).

- ・ 回 ト ・ モト ・ モト

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

A (1) > A (1) > A

Introduction

- Xenopus and its Lateral Line
- Water Waves

2 Minimal Model

- Reconstruction of the Wave Form
- Performance of the Minimal Model
- Learning
- 3 Field Reconstruction
 - Full-Field Reconstruction
 - Window of Reference

4 Neuronal Implementation of the Model

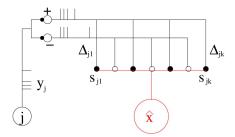
- Implementing a Convolution
- Learning a Neuronal Model
- Performance of a Neuronal Model

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

Neuronal Implementation of a Convolution

$$\hat{x}(t-T) = \sum_{jk} s_{jk} y_i \left(t - \Delta_{jk}\right) \tag{21}$$

イロト イヨト イヨト イヨト



Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

Basic Learning Equation

Synaptic learning with a learning window $W(t_2 - t_1)$:

$$\Delta J_{ik} = \eta \int dt_1 \int dt_2 W (t_2 - t_1) \Sigma_{ik}^{\text{in}} (t_2) \Sigma^{\text{out}} (t_1) .$$
 (22)

Choose $W(t) = -2 \epsilon(-t)$ and make sure that $\Sigma^{out} = \hat{x}(t) - x(t)$ by giving x(t) as additional inhibitory input to ensure equivalence to our learning equation

$$\Delta s_{ik} = \mathcal{F}^{-1} \left[-2\eta \left(\hat{X} - X \right) Y_i^* \right] .$$
⁽²³⁾

(日) (部) (注) (注)

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

Reverse Transfer Functions I

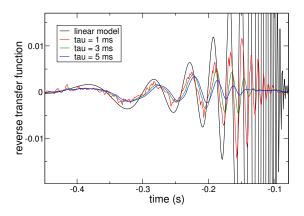


Figure: Reverse transfer functions implemented in neuronal hardware. The linear model can only provide rough comparison.

A (10) < 10</p>

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

・ロト ・ 一下・ ・ 三ト・・

Reverse Transfer Functions I

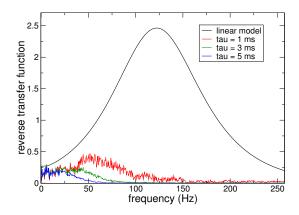


Figure: Same as the last slide in frequency space. The broadening of a spike in the PSPs acts as a low pass filter.

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

Localisation

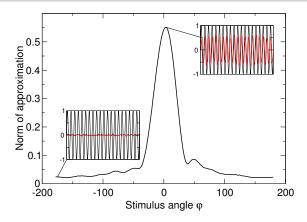


Figure: Localisation performance of a field of 180 neurons after 25000 learning steps.

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

イロト イヨト イヨト イヨト

3



What still has to be done:

• convergence of the neuronal model

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

イロト イヨト イヨト イヨト

æ



What still has to be done:

- convergence of the neuronal model
- distance learning

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

イロト イヨト イヨト イヨト

æ



What still has to be done:

- convergence of the neuronal model
- distance learning
- underwater detection

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

(日) (日) (日) (日) (日)

æ

Outlook

What still has to be done:

- convergence of the neuronal model
- distance learning
- underwater detection
- reference input

Implementing a Convolution Learning a Neuronal Model Performance of a Neuronal Model

イロト イポト イヨト イヨト

Bibliography



A. Elepfandt and L. Wiedemer.

Lateral-line responses to water surface waves in the clawed frog, *Xenopus laevis*.

J. Comp. Physiol. A, 160:667-682, 1987.

B. Claas and H. Münz.

Analysis of surface wave direction by the lateral line system of *Xenopus*.

J. Comp. Physiol., 178(2):253-268, 1996.



J. M. P. Franosch, M. C. Sobotka, A. Elepfandt, and J. L. van Hemmen.

Minimal model of prey localization through the lateral-line system. *Physical Review Letters*, 91(15), October 2003.