## Digital Signal Processing - SS15 Theory Tutorial 4

## Fourier Transform

PROBLEM ONE Consider the Fourier transform pair

$$
\mathrm{e}^{-|t|} \xrightarrow{F} \frac{2}{1+\omega^{2}}
$$

(a) Use the appropriate Fourier transform properties to find the Fourier transform of $t e^{-|t|}$.
(b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$
\frac{4 \mathrm{t}}{\left(1+t^{2}\right)^{2}}
$$

PROBLEM TWO Let $x(t)$ be signal with Fourier transform $X(j \omega)$. Suppose we are given the following facts:

1. $x(t)$ is real.
2. $x(t)=0$ for $t \leqslant 0$
3. $\quad \frac{1}{2 \pi} \sum_{-\infty}^{\infty} \mathfrak{R}\{X(j \omega)\} \mathrm{e}^{j \omega t} d \omega=|t| \mathrm{e}^{-|t|}$

Determine a closed-form expression for $x(t)$.

PROBLEM THREE Determine whether each of the following statements is true of false. Justify your answers.
(a) An odd and imaginary signal always has an odd and imaginary Fourier transform.
(b) The convolution of an odd transform with an even Fourier transform is always odd.

