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Consider the travelling salesman problem (TSP)

- Minimal spanning tree (MST) heuristic: If partial tour already exists, MST of a subset of cities is the minimal sum of the connection cost of any *tree* connecting these cities
- a) Show that this heuristic can be derived from a relaxed version of the TSP

The TSP is the search for the cheapest closed loop to visit all given cities. Relaxing this without the need to close the loop, a minimal spanning tree is a relaxed sub-problem which connects all cities with minimal connection cost. The MST heuristic is admissible as it is always shorter than or equal to a closed loop.



- b) Show that this heuristic dominates the straight-line distance MST dominates the straight-lines distance means that it has always greater or equal values: either there is a direct (straight) connection, or it includes 2 or more lines where the triangle inequality leads to greater values.
- c) Are there efficient ways to solve the MST problem? Yes, see e.g. http://homes.cs.washington.edu/~jinna/ugrad_work/thesis.pdf



Consider the following game



- A moves first
- Each player must move to emtpy field in either direction
- If field occupied, the player can jump over the opponent, if there is a field
- Goal: reach the start field of the opponent
- Utility function for A: +1, if A reaches field 4 first - 1, if B reaches field 1 first



a) Draw the game tree using state (field for A, field for B)





- a) Mark the nodes with their MINIMAX values How are the "?" handled? Why? Minmax values are shown in blue on the previous slide If all successors are "?" then the value is "?", otherwise the "?" is ignored when calculating the min or max value
- b) Why does the Minimax-algorithms fail for that tree? How can the tree be repaired? Using depth-first search, it will run into an infinite loop. Keeping track of previous states to identify loops would help in that case.



a) How many Tic Tac Toe games are possible?



An upper bound of final states is 9! if the order of placing the marks is taken into account. Goal states (win situations for one of the players) without filling th whole board need to be subtracted. For different game states ignoring the path to the state the upper bound is 3⁹, as each position can be empty or be filled with either a X or a O.



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 b) Draw a tree with 2 levels and show the utility values at level 2 (you can ignore positions resulting from symmetry operations on the board)
Level 0 is the empty field, X starts. At level 1, there are

only 3 possibilities. At level 2, there are 12 possibilities



c) Use the minimax algorithm to choose the best initial move Use the evaluation function eval(s) = 1 if there is a row, column, or diagonal with 3 "X", eval(s) = -1 if there are 3 "O". Other terminal states have the value of 0. Let X_n be the numbers of rows, columns, or diagonals with exactly n "X" and no "O". O_n is defined analoguously. If the state is not a terminal state, then $eval(s) = (3 X_2 + X_1) - (3 O_2 + O_1)$

For the minimax values, see next slide

d) Show the nodes *not* expanded because of pruning

see next slide, the nodes not expanded are marked red





